Parents determine the traits of their offspring via both genetic inheritance and through their own traits. A simple model of this (Kirkpatrick & Lande 1989) assumes an individual's trait value is $z = a + e + mz'$, where a is the influence of genetics, e is the influence of environment, and mz' is the influence of their parent's trait (assuming only one parent's trait influences the offspring – this is typically referred to as 'maternal effects' but equally applies to fathers). Here z' is their parent's trait value and $0 \leq m \leq 1$ is the proportion of the parent's trait that is passed down. To give a concrete example, the trait value could be size, and larger parents produce larger offspring regardless of genetics.

Assuming stabilizing selection around an optimal trait value of 0, this model produces the following recursions for the mean genetic value, $\bar{a}$, and mean trait value, $\bar{z}$, in generation $t + 1$,

$$\begin{aligned}\bar{a}(t+1) &= \bar{a}(t) - k_g \bar{z}(t) \\ \bar{z}(t+1) &= \bar{a}(t) + (m(1-k_p) - k_g)\bar{z}(t)\end{aligned}$$

where $0 \leq k_g < 1$ is the evolutionary potential of the genetic value and $0 \leq k_p < 1$ is the evolutionary potential of the trait value.

1. [4/20 points] We can write this system of equations in matrix form, $\vec{x}(t+1) = \mathbf{M}\vec{x}(t)$, where $\vec{x}(t) = \begin{pmatrix} \bar{a}(t) \\ \bar{z}(t) \end{pmatrix}$ and $\mathbf{M}$ is a matrix of parameters (m, k_g, k_p). What are the entries of $\mathbf{M}$?

2. [4/20 points] Find the equilibrium of this model, $\hat{\vec{x}}$.

3. [4/20 points] To determine the stability of $\hat{\vec{x}}$, calculate the eigenvalues of $\mathbf{M}$.

4. [4/20 points] Determine the conditions under which the dynamics will be oscillatory.

5. [4/20 points] Given the dynamics are oscillatory, show that the equilibrium is stable if $m(1 - k_p) < 1$.